LOCAL ANALYSIS OF AN MHD GENERATOR WITH A T LAYER

V. V. Ovchinnikov and V. S. Slavin UDC 538.4

One-dimensional magnetohydrodynamic models have been used [1-3] to examine the self-maintaining current layer (T layer) in a flow of ionized gas in a transverse magnetic field. It has been found that the occurrence of the T layer accentuates the interaction of the flow with the field, which can be used in an efficient MHD generator. The theoretical possibility of making such an electricity generator has been demonstrated in [4], in which a one-dimensional model was used to calculate an arbitrarily selected generator state. However, the model was complicated and took the form of a system of partial differential equations, so it is very difficult to use it to analyze the operation of T-layer MHD generators. One can use the available information on the gasdynamic processes in a flow containing a T layer [5] in an elementary theory of the MHDG containing a T layer, where the layer is represented by an impermeable and nondeformable piston. This approach provides a relation between the magnetic-field induction in the channel, the expansion of the working part in relation to the critical section, the retardation parameters of the working body at the inlet, and the temperature and velocity of the T layer. The result is parametrically dependent on the thickness of the T layer, which remains undetermined. To avoid this indeterminacy, it is necessary to consider the physical processes in the T layer that are responsible for its structure and definite dimensions. This problem has been considered in [6] for a stationary T-layer structure stabilized by thermal-conduction heat loss. However, in an MHD generator in which the characteristic length of the channel is $l^* \sim 10$ and the characteristic velocity is $u^* \sim 10^3$ m/sec, the time to reach this stationary solution is $\tau_\lambda \gg \tau^* = l^*/u^* \sim 10^{-2}$ sec.

We consider the processes in the channel of an MHD generator with solid electrodes (Fig. 1, where D is the shock-wave speed). The working body is not electrically conducting and after expansion in the accelerating nozzle moves with a supersonic velocity. The flow introduces an isobaric temperature perturbation into the working part of the channel, and this produces electrical conductivity and interaction with the magnetic field. The magnetohydrodynamic process produces the self-maintaining current layer from the temperature perturbation, whose structure is found by solving a system of magnetic gasdynamic equations in Lagrange variables, where it is assumed that $B_{ind}/B_0 \approx R_m(1-k) = \mu_0 \sigma u \delta (1-k) \ll 1$, which enables one to neglect the induced magnetic field, i.e., $B_{ind} = 0$, $\partial E/\partial x = 0$:

$$d\rho^{-1}/dt - \partial v/\partial s = 0; \tag{1}$$

$$dx/dt = v; \tag{2}$$

$$\rho dv/dt = -\rho \partial p/\partial s + B_0 j; \tag{3}$$

$$c_V \rho dT/dt - RT d\rho/dt = j^2/\sigma - q_{em}; \tag{4}$$

$$\frac{\partial}{\partial s}(j/\sigma) = -B_0 \frac{\partial v}{\partial s}; \tag{5}$$

$$j = -\rho \partial \psi/\partial s; \tag{6}$$

$$p = R\rho T; \tag{7}$$

$$\sigma = \sigma_0 (T/T_c)^n. \tag{8}$$

Here v is the velocity of the relative motion of the various parts of the T layer. The velocity field equalizes when force balance is attained and v = 0. The working body is assumed to be an ideal gas with adiabatic parameter $\gamma = 1.2$ and molecular mass $\mu = 30$, which represents closely the combustion products with the approximate composition 25% CO_2 +5% H_2O +70% N_2. System (1)-(8) can be simplified if one bears in mind that at the parameters of the process $p \sim 10^5$ N/m^2, $T \sim 2 \cdot 10^3$K, $T_c \sim 10^4$K, $\sigma \sim 10^3$ mho/m, $u \sim 10^3$ m/sec, $B \sim 2$T, $\delta_0 \sim 10^{-1}$ m (the initial dimension of the T layer) the characteristic times are related by

$$\tau_F \ll \tau_Q < \tau^*, \tag{9}$$

Krasnoyarsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 26-34, July-August 1983. Original article submitted January 1, 1982.

 0021-8944/83/2404-0468$07.50

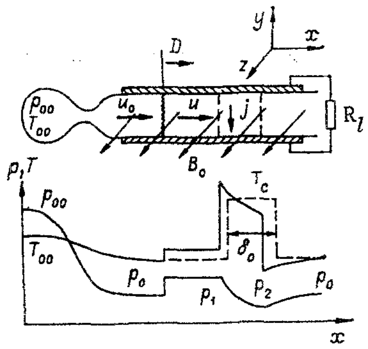

Fig. 1

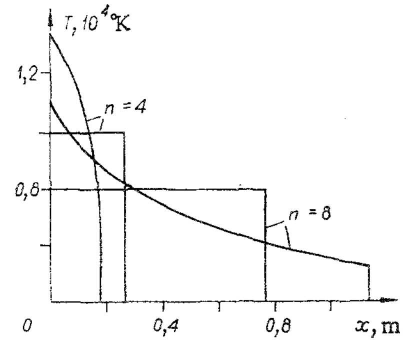

Fig. 2

where $\tau_F = \delta_0/\sqrt{\gamma R T_c} \sim \rho/\sigma B^2 \sim 10^{-5} - 10^{-4}$ sec is the time required to establish force balance in the T layer, and the order of this can be determined by evaluating the terms in the equation of motion (3), while $\tau_Q = c_V \rho T_c/\sigma u^2 B^2 \sim 10^{-3}$ sec is the time required to establish thermal balance.

This means that one can split up the task into two stages. In the first, one determines the structure of the T layer formed by adiabatic action of bulk electrodynamic forces. Some of the mass of gas in the T layer is compressed and correspondingly heated, while the rest cools on adiabatic expansion. In the second stage we solve energy equation (4), in which the sign of the right side $j^2/\sigma - q_{ind}$ determines the direction of the process.

We then have to solve the following system of dimensionless equations, in which we neglect the convective motion and the nonadiabatic effects of Joule dissipation and radiation:

$$\partial p/\partial s + \partial\psi/\partial s = 0; \tag{10}$$

$$\rho dT/dt = (\gamma - 1)T d\rho/dt; \tag{11}$$

$$\frac{\partial}{\partial s}\left(\frac{\rho}{\sigma}\frac{\partial\psi}{\partial s}\right) = 0; \tag{12}$$

$$\sigma = T^n; \tag{13}$$

$$p = \rho T; \tag{14}$$

$$\partial x/\partial s = \rho^{-1}. \tag{15}$$

The scales for the dimensionless quantities are as follows: p_1 the pressure behind the shock front reflected from the T layer (Fig. 1), T_c the initial temperature of the T layer, σ_0 the coefficient from the approximating expression (8), and δ_0 the initial dimension of the T layer. As the initial conditions, one specifies an isobaric temperature perturbation for the dimension δ_0 with parameters $T(s, t = 0) = T_c$, $p(s, t = 0) = p_0$. The pressure distribution for $t \geqslant \tau_F$ is found by solving the stationary problem with the boundary conditions $p(s = 0) = 1$, $x(s = 0) = 0$, $p(s = 1) = \Delta$, where $\Delta = p_2/p_1$ is a characteristic parameter.

The solution is represented in the form

$$\psi = 1 - p; \tag{16}$$

$$p = [1 - (1 - \Delta^\beta)s]^{1/\beta}; \tag{17}$$

$$x = \frac{p_0^{1/\gamma}\beta\gamma}{(1 - \Delta^\beta)(\beta\gamma - 1)}\left[1 - p^{\frac{\beta\gamma - 1}{\gamma}}\right]; \tag{18}$$

$$T = (p/\rho_0)^{(\gamma-1)/\gamma}, \tag{19}$$

where $\beta = [(\gamma + 1) - n(\gamma - 1)]/\gamma$.

Figure 2 gives the characteristic temperature profiles. The solution is dependent on the degree of nonlinearity n in the equation $\sigma = T^n$. Both states have been selected from the conditions $\sigma_1\delta_{01} = \sigma_2\delta_{02}$, with $n = 4$ describing $\sigma(T)$ in the range $9\cdot10^3$ °K $\leqslant T \leqslant 11\cdot10^3$ °K and $n = 8$ for the range $7\cdot10^3$ °K $\leqslant T \leqslant 9\cdot10^3$ °K.

The conditions at the boundary of the T layer (p_1, p_2, and velocity u) in turn are determined by the set of independent parameters, which taken together uniquely characterize the generator process. These are the following quantities: p_{00}, the pressure in the combustion chamber; T_{00}, the temperature in the combustion chamber; c_p and c_V, the specific heats of

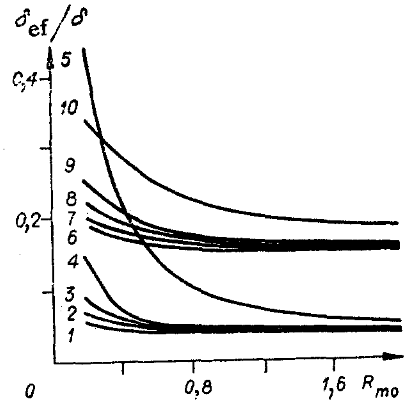

Fig. 3

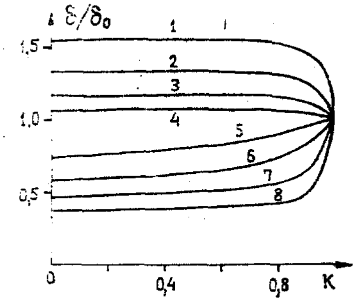

Fig. 4

the working body; $\varepsilon(T, p, \delta)$, the degree of blackness in the plasma layer with temperature T, pressure p, and dimension δ; A_*, the critical section of the channel; A, the section in the working part; T_c, the initial temperature in the T layer; δ_0, the initial dimensions of the T layer; B_0, the induction of the external magnetic field; and R_l, the load resistance.

From this set of dimensional parameters we can derive dimensionless combinations characterizing the process: $N = B_0^2/2\mu_0 p_{00}$, the ratio of the magnetic pressure to the pressure in the combustion chamber, $R_{m0} = \mu_0\sigma_0\delta_0 c_{00}$, the analog of the magnetic Reynolds number, which is determined via the speed of sound in the combustion chamber c_{00}, $\gamma = c_p/c_v$, the adiabatic parameter, and $M_0 = u_0/c_0$, the Mach number for the unperturbed flow in the working part of the channel, which is related to the degree of expansion by

$$A/A_* = [2/(\gamma+1)]^{(\gamma+1)/2(\gamma-1)}\left(1+\frac{\gamma-1}{2}M_0^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}\Big/M_0,$$

together with $K = E/uB_0$, the load parameter (we assume that K is given and that the load resistance can always be selected), and $\sum_{00} = \dfrac{4\varepsilon_0\sigma_{SB}T_c^4}{\sigma_0 c_{00}^2 B_0^2\delta_0}$, a dimensionless parameter defining the ratio of the radiative loss from a particular working body to the energy performance of the MHD channel (here the degree of blackness ε_0 is defined as $\varepsilon(T_c, p_{00}, \delta_0)$).

For definiteness we will consider states with $p_{00} = 10^6$ Pa, $T_{00} = 3000$ K, $B_0 = 2$T, $T_c = 10^4$ K, $\sigma_0 = 3\cdot10^3$ mho/m, $\delta_0 = 0.25$ m; then $N = 1.6$; $R_{m0} = 1$; $\gamma = 1.2$. These states differ in degree of expansion in the channel, for which M_0 is correspondingly 1.5, 2, 2.5, and 3. The load parameter then will vary in the range $0 \leqslant K \leqslant 1$.

We now write equations relating the conditions at the boundary of the T layer to the dimensionless parameters of the MHD generator:

$$p_1 = p_0\left(\frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1}\right); \tag{20}$$

$$p_2 = p_0(c_2/c_0)^{2\gamma/(\gamma-1)}, \tag{21}$$

where $M_1 = (u_0 - D)/c_0$ is the shock-wave Mach number and $c_2 = c_0 - [(\gamma-1)/2](u_0 - u)$ is the speed of sound in the negative-pressure wave (u is the speed of the T layer). The parameters with subscript zero correspond to those of the unperturbed flow, which in turn are defined by the following:

$$p_0 = p_{00}\left(1+\frac{\gamma-1}{2}M_0^2\right)^{-\gamma/(\gamma-1)}, \quad T_0 = T_{00}\left(1+\frac{\gamma-1}{2}M_0^2\right)^{-1},$$
$$c_0 = (\gamma R T_0)^{1/2}, \ u_0 = c_0 M_0. \tag{22}$$

The speed of the shock-wave front is determined from the equation of continuity:

$$\frac{u_0 - D}{u - D} = \frac{\rho}{\rho_0} = \frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2} = \frac{\frac{\gamma+1}{2}\left(\frac{u_0-D}{c_0}\right)^2}{1+\frac{\gamma-1}{2}\left(\frac{u_0-D}{c_0}\right)^2}. \tag{23}$$

We introduce the parameter $\lambda = u/c_0$, which is the dimensionless velocity of the T layer. Then from (20)-(23) we find the pressure ratio:

$$f(M_0, \lambda) = \frac{p_1}{p_0} = \frac{2\gamma}{\gamma+1}\left\{\frac{\gamma+1}{4}(M_0-\lambda)+\left[\left(\frac{\gamma+1}{4}(M_0-\lambda)\right)^2+1\right]^{1/2}\right\}^2-\frac{\gamma-1}{\gamma+1}. \quad (24)$$

Under conditions of force balance we have

$$\frac{dp}{dx} = jB_0 = -\sigma(x)uB_0^2(1-K)$$

or in integral form

$$p_1 - p_2 = uB_0^2(1-K)\int_0^\delta \sigma(x)\,dx. \quad (25)$$

We determine the dimensionless velocity from this equation using (20)-(23):

$$\lambda = \frac{(1-\Delta)f(M_0, \lambda)\left(1+\frac{\gamma-1}{2}M_0^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}{2R_{m0}N(1-K)\Delta_\sigma\Delta_\delta}, \quad (26)$$

where

$$\Delta_\sigma = \int_0^\delta \sigma(x)\,dx/\sigma_0\delta, \quad \Delta_\delta = \delta/\delta_0. \quad (27)$$

We determine $\Delta = p_2/p_1$ from (20) and (21) to get

$$\Delta = \left[1-\frac{\gamma-1}{2}(M_0-\lambda)\right]^{2\gamma/(\gamma-1)}\Big/ f(M_0, \lambda). \quad (28)$$

We use the solution of (17)-(19) for the T layer to express the integrals of (27) in terms of the flow parameters:

$$\Delta_\sigma = \frac{\beta\gamma f^{-\frac{(\gamma-1)n}{\gamma}}}{(1-\Delta^\beta)(\gamma+1)}\left[1-\Delta^{\frac{\gamma+1}{\gamma}}\right];$$

$$\Delta_\delta = \frac{f^{-1/\gamma}\beta\gamma}{(1-\Delta^\beta)(\beta\gamma-1)}\left[1-\Delta^{\frac{\beta\gamma-1}{\gamma}}\right]. \quad (30)$$

Then equations (24), (17)-(19), (26), (30) in inexplicit form relate the solution in the T layer to the parameters of the MHD generator. By solving these equations numerically one can establish the dependence of Δ and λ on the following set of dimensionless characteristics: M_0, N, R_{m0}, K, γ, n.

A very important characteristic of an MHD generator as a heat engine is the parameter η_N, the degree of conversion of the gas enthalpy into electrical energy, which for an MHD generator with a T layer is defined as

$$\eta_N = \frac{(p_1-p_2)uAK}{\rho_* u_*\left(c_pT_* + u_*^2/2\right)A_*}. \quad (31)$$

On converting to dimensionless variables, (31) is rewritten as

$$\eta_N = \frac{\gamma-1}{\gamma}\,\frac{\lambda(1-\Delta)K}{M_0}\,\frac{f(M_0, \lambda)}{1+\frac{\gamma-1}{2}M_0^2}.$$

Figures 3-5 give the result from analyzing the working conditions in a T-layer MHD generator.

When the T layer is formed from an initial perturbation with dimension δ_0, part of the heated gas cools on adiabatic expansion and thereby loses its electrical conductivity and will not interact with the magnetic field. On the other hand, the more extended T layer enables one to pass a large current and therefore to provide a large degree of conversion. One then has to consider the optimum dimensions of the initial perturbation. We determine the effective size δ_{ef} of the T layer as the cross section carrying half of the total current:

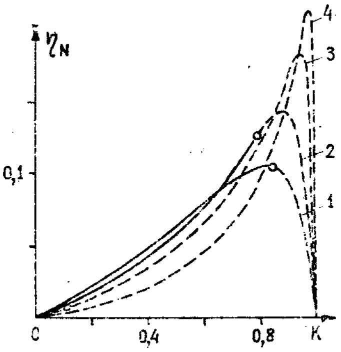

Fig. 5

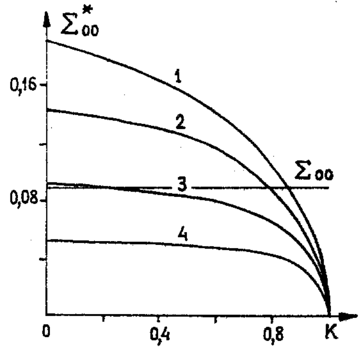

Fig. 6

$$\int_0^{\delta_{ef}} j(x)\,dx = I/2.$$

This equation can be solved for δ_{ef} on the basis of (17)-(19):

$$\delta_{ef} = \delta\{1 - [1 - (1 - \Delta)^{\beta}][1 - (0.5(1 + \Delta))^{\beta}]/(1 - \Delta^{\beta})\}^{(\beta\gamma-1)/\beta\gamma}/(1 - \Delta^{(\beta\gamma-1)/\gamma}).$$

Figure 3 gives the dependence of the dimensionless thickness δ_{ef}/δ of the T layer on R_{mo}, i.e., in fact on δ_0. As R_{mo} increases, δ_{ef}/δ at first decreases sharply but then stabilizes in the state with $n_1 = 4$ at the level of about 0.2 (K = 0.1, 0.3, 0.5, 0.7, 0.9 for lines 6-10 correspondingly), while in the state with $n_2 = 8$ it settles at the level of about 0.02 (K = 0.1, 0.3, 0.5, 0.7, 0.9 for lines 1-5 correspondingly). This difference is due to a qualitative difference in the temperature distributions in the T layer. The convex profile for $n_1 = 4$ sets up a condition for practically homogeneous current flow throughout the volume of the plasma, i.e., all the initially heated gas operates under this condition. In the case $n_2 = 8$, the stabilization of the 0.02 level is explained by the presence of a weak background conductivity at low temperatures. For this state we conclude that the current in the plasma is concentrated in a region adjoining the left boundary of the T layer, whose dimensions cease to be dependent on the dimensions of the initial perturbation. Here the rest of the mass of gas in the T layer ceases to interact with the magnetic field and falls under the influence of the negative-pressure wave, where it expands and increases the overall dimension δ. The confirmation of this is provided by the character of the δ/δ_0 dependence in the states with $n_1 = 4$ (Fig. 4, $M_0 = 3$, 2.5, 2, and 1.5 for lines 1-4 correspondingly) and with $n_2 = 8$ ($M_0 = 3$, 2.5, 2, and 1.5 for lines 5-8 correspondingly). Consequently, the adiabatic effect of the electrodynamic bulk force leads to contraction in the current zone. Naturally, this process will be accentuated in the following stage of the thermal stabilization, where the additional Joule heating of the current-concentration regions leads to the familiar phenomenon of thermal contraction. The states with n_1 and n_2 differ in the sign of the derivative in the solution for T(x). In the range $n_1 < n^* < n_2$ we can give a value $n^* = 1/(\gamma - 1)$ for which the solution T(x) is a linear function and $\delta/\delta_0 = 1$. Therefore, a rise in temperature in the initial perturbation, which leads to obedience to the condition $n > n^*$, enables one to produce a homogeneous current layer, in which all the mass of the initially heated gas operates, or conversely for initial conditions such that $n < n^*$, part of the mass of the T layer is discarded and the energy consumed in heating it is lost. On the basis of the results in Fig. 3, the parameters of the initial perturbation should be chosen such as to obey $n < n^*$ and $R_{m0} \leqslant 0.5$.

Figure 5 shows $\eta_N(K)$ in the state with n = 4 for $M_0 = 1.5$, 2, 2.5, and 3 (lines 1-4 correspondingly). As any change in the load is directly reflected in the gas dynamics of the flow containing the T layer, the η_N dependence is not a symmetrical parabola with its maximum at K = 0.5. The dependence of η_N on M_0 is interesting. In the region of practically significant values for the load parameter ($0.5 \leqslant K \leqslant 0.8$), the maximum degree of conversion is obtained when M_0 is in the range $1.5 \leqslant M \leqslant 2$; for values $M_0 > 2$, it might appear that one could increase η_N, but it will be shown below that failure to obey the energy-balance conditions makes these states unstable (they are shown by the broken parts of the curves in Fig. 5).

We now consider the stability of a T-layer MHD generator. When the electrodynamic force has produced a certain temperature distribution T(x) in the T layer, an energy mechanism is activated that will determine whether the temperature rises or falls in accordance with the sign of the right side in (4), $j^2/\sigma - q_{ind}$. Here the construction of the temperature is accompanied by change in the pressure, which by virtue of the condition $\tau_F \ll \tau_Q$ is described by the equation

$$\partial p/\partial x = jB_0 = -\sigma(T)(1-K)uB_0^2.$$

The overall power dissipated in the T layer is determined as the integral $Q_{jo} = \int_0^\delta \sigma(T)[1 - K]^2 u^2 B_0^2 dx$, where K, u, and B_0 are independent of x, so consequently

$$Q_{jo} \sim \Omega = \int_0^\delta \sigma(T)\,dx = \sigma_0 \delta \Delta_\sigma.$$

If the dynamics of the process are such that $d\Omega/dt \geqslant 0$, the T layer will be considered as stable and by the time $t \sim \tau_Q$ a stationary structure will be established in it, while otherwise the T layer will be unstable and should split up.

To determine the bulk radiative energy loss, we use a very simple model for a radiating homogeneous layer, for which $q_{ind} = 4\varepsilon\sigma_{SB}T^4/\delta$. Here $\varepsilon(T, p, \delta)$ is the emissivity of a hemispherical gas volume with composition $90\%CO_2 + 10\%N_2$ [7]. The coefficient 4 appears in converting from a hemisphere to a cube radiating on all faces. Naturally, such a crude model can provide only qualitative results.

We therefore solve the following system of dimensionless equations with the same set of boundary conditions as for (10)-(15):

$$\partial p/\partial s + \partial\psi/\partial s = 0; \quad (32)$$

$$\rho\frac{dT}{dt} - (\gamma-1)T\frac{d\rho}{dt} = \omega\left[\frac{\rho^2}{\sigma}\left(\frac{\partial\psi}{\partial s}\right)^2 - \sum T^4\right]; \quad (33)$$

$$\frac{\partial}{\partial s}\left(\frac{\rho}{\sigma}\frac{\partial\psi}{\partial s}\right) = 0; \quad (34)$$

$$\partial x/\partial s = \rho^{-1}; \quad (35)$$

$$\sigma = T^n, \ p = \rho T. \quad (36)$$

The dimensionless parameter ω is defined as $\omega = \tau/\tau_Q$, where τ is the characteristic time. We assume that $\tau_F \leqslant \tau \ll \tau_Q$, i.e., $\omega < 1$. The dimensionless group

$$\sum = \frac{4\varepsilon\sigma_{SB}T_c^4}{\langle\sigma\rangle u^2 B_0^2 (1-K)^2 \delta} \quad (37)$$

is a dynamic characteristic, since it is dependent on the dynamic and local parameters $\varepsilon(T_c, \langle p\rangle, \delta)$, $\langle\sigma(x)\rangle$, u, δ.

In the T layer, $\varepsilon < 0.1$, which is characteristic of a bulk emitter. Therefore, we can assume a linear relationship:

$$\varepsilon = \varepsilon_0 \frac{\langle p\rangle}{p_{00}} \frac{\delta}{\delta_0} = \varepsilon_0 \Delta_p \Delta_\delta, \quad (38)$$

where $\varepsilon_0 = \varepsilon(T_c, p_{00}, \delta_0)$, and the other parameters have been defined above:

$$u = \lambda c_0 = \lambda c_{00}\left(1 + \frac{\gamma-1}{2}M_0^2\right)^{-1/2}, \quad \sigma = \sigma_0\Delta_\sigma, \quad \delta = \delta_0\Delta_\delta.$$

Equation (37) can be rewritten as

$$\sum = \sum\nolimits_{00} \frac{\Delta_p (1-\Delta)^2\left(1 + \frac{\gamma-1}{2}M_0^2\right)}{\Delta_\delta^2 \Delta_\sigma^2 \lambda^2 (1-K)^2}.$$

We use (32) and the equation of state (36) to rewrite (32)-(35):

$$\frac{\gamma}{\gamma-1}\frac{p}{T}\frac{dT}{dt} - \frac{dp}{dt} = \omega\left[\frac{p^2}{T^{n+2}}\left(\frac{\partial p}{\partial s}\right)^2 - \sum T^4\right]; \quad (39)$$

$$\frac{\partial}{\partial s}\left(\frac{p}{T^{n+1}}\frac{\partial p}{\partial s}\right) = 0; \quad (40)$$

$$\partial x/\partial s = \rho^{-1}. \tag{41}$$

By solving (39)-(41) we can determine the values of the following functional for each set of parameters R_{mo}, N, K, γ, n, Σ_{oo}:

$$\Omega(t) = \int_0^\delta \sigma(T)\,dx.$$

Correspondingly, from the condition $d\Omega^*/dt = 0$ we get a certain critical value of Σ_{00}^*, and then if the real value is $\Sigma_{00} \leqslant \Sigma_{00}^*$ the state is stable.

To solve (39)-(41) we use the method of expansion with respect to the small parameter ω and restrict ourselves to the first correction:

$$T = \tilde{T}_0 + \omega\tilde{T}_1,\ p = \tilde{p}_0 + \omega\tilde{p}_1,\ \Sigma = \Sigma_0 + \omega\Sigma_1,$$
$$\lambda = \lambda_0 + \omega\lambda_1,\quad \Delta = \Delta_0 + \omega\Delta_1,\ \Delta_\delta = \Delta_{\delta_0} + \omega\Delta_{\delta_1},$$
$$\Delta_\sigma = \Delta_{\sigma_0} + \omega\Delta_{\sigma_1},\quad \Delta_p = \Delta_p + \omega\Delta_{p_1}.$$

Here the subscript 0 denotes the zeroth (adiabatic) solution of (16)-(19), while subscript 1 denotes the first approximation to the solution of (39)-(41).

The equations for the first approximation take the form

$$\frac{\gamma}{\gamma-1}\frac{\tilde{p}_0}{\tilde{T}_0}\frac{d\tilde{T}_1}{dt} - \frac{d\tilde{p}_1}{dt} = \frac{\tilde{p}_0^2}{\tilde{T}_0^{n+2}}\left(\frac{\partial\tilde{p}_0}{\partial s}\right)^2 - \sum\nolimits_0 \tilde{T}_0^4; \tag{42}$$

$$\frac{\partial}{\partial s}\left[\frac{\tilde{p}_1}{\tilde{T}_0^{n+1}}\frac{\partial\tilde{p}_0}{\partial s} + \frac{\tilde{p}_0}{\tilde{T}_0^{n+1}}\frac{\partial\tilde{p}_1}{\partial s} - \frac{\tilde{p}_0\tilde{T}_1}{\tilde{T}_0^{n+2}}\frac{\partial\tilde{p}_0}{\partial s}\right] = 0. \tag{43}$$

Initial conditions: $\tilde{T}_1(s, t=0) = \tilde{p}_1(s, t=0) = 0$. Boundary conditions: $\tilde{p}_1(s=0, t) = 1$, $\tilde{p}_1(s=1, t) = \Delta_1$.

We use a Laplace transformation to solve (42) and (43) and after cumbersome but obvious operations we get the solution for $\tilde{T}_1(s, t)$. Then we solve the condition $\frac{d\Omega^*}{dt} = n\int_0^1 \tilde{T}_0^{n-1}\tilde{T}_1\,ds = 0$ for Σ_{00}^* to get

$$\Sigma_{00}^* = \frac{\Delta_{\sigma_0}\lambda_0^2(1-K)^2\left[d_6 I_6 - \frac{\gamma-1}{\gamma}(d_1 I_{13} - d_3 I_{14})\right]}{\Delta_{\delta_0} f(M_0, \lambda_0)\left(1 + \frac{\gamma-1}{2}M_0^2\right)^{-1/(\gamma-1)}\left[d_5 I_3 - \frac{\gamma-1}{\gamma}(d_2 I_{13} - d_4 I_{14})\right]},$$

where

$$d_1 = a_2\frac{1 - \Delta_0^{\frac{n(\gamma-1)+1}{\gamma}}}{1 - \Delta_0^{\frac{\gamma+1}{\gamma}}};\quad d_2 = a_3\frac{1 - \Delta_0^{\frac{n(\gamma-1)+1}{\gamma}}}{1 - \Delta_0^{\frac{\gamma+1}{\gamma}}};$$

$$d_3 = a_2\frac{\Delta_0^{\frac{\gamma+1}{\gamma}} - \Delta_0^{\frac{n(\gamma-1)+1}{\gamma}}}{1 - \Delta_0^{\frac{\gamma+1}{\gamma}}};\quad d_4 = a_3\frac{\Delta_0^{\frac{\gamma+1}{\gamma}} - \Delta_0^{\frac{n(\gamma-1)+1}{\gamma}}}{1 - \Delta_0^{\frac{\gamma+1}{\gamma}}};$$

$$d_5 = \frac{5\gamma-4}{4\gamma-3} f(M_0, \lambda_0)^{\frac{4(\gamma-1)}{\gamma}};$$

$$d_6 = \frac{\left(1 - \Delta_0^\beta\right)^2[n(\gamma-1)+\gamma]}{\beta^2[n(\gamma-1)+1]\, f(M_0, \lambda_0)^{\frac{(n+2)(\gamma-1)}{\gamma}}};$$

$$a_2 = \frac{\left(1 - \Delta_0^\beta\right)^2\gamma}{\beta^2[n(\gamma-1)+1]\, f(M_0, \lambda_0)^{\frac{(n+2)(\gamma-1)}{\gamma}}};$$

$$a_3 = \frac{\gamma}{4\gamma - 3} f(M_0, \lambda_0)^{\frac{4(\gamma-1)}{\gamma}};$$

$$I_6 = \frac{\beta\gamma\left(1 - \Delta_0^{\frac{n(\gamma-1)+1}{\gamma}}\right)}{\left(1 - \Delta_0^{\beta}\right)[n(\gamma - 1) + 1]}; \quad I_3 = \frac{\beta\gamma\left(1 - \Delta_0^{\frac{4\gamma-3}{\gamma}}\right)}{(4\gamma - 3)\left(1 - \Delta_0^{\beta}\right)};$$

$$I_{13} = \frac{\beta\gamma\left(1 - \Delta_0^{\frac{\gamma+1}{\gamma}}\right)}{(\gamma + 1)\left(1 - \Delta_0^{\beta}\right)}; \quad I_{14} = -\frac{\beta \ln \Delta_0 f(M_0, \lambda)^2}{\left(1 - \Delta_0^{\beta}\right)}.$$

Figure 6 shows the dependence of $\sum_{00}^{*}$ on the load parameter for various values of the Mach number (M_0 =1.5, 2, 2.5, 3 for lines 1-4 correspondingly) when N =1.6, R_{m0} =1, n =4, γ =1.2, and it also shows the value of the real parameter Σ_{00}. The points of intersection of the Σ_{00} straight line with the curves represent the boundary separating the region of stable states from the unstable ones.

Therefore, we have proposed a method of local analysis for a T-layer MHD generator which resembles the local analysis for an MHD generator with a continuous flow [8] in enabling one to select the optimum modes of operation. The preliminary results show that the degree of conversion of the enthalpy in the combustion products into electrical energy in one T layer may be about 10%.

LITERATURE CITED

1. A. N. Tikhonov, A. A. Samarskii, et al., "The nonlinear effect of formation of a self-maintaining hot electrically conducting gas layer in nonstationary processes in magnetohydrodynamics," Dokl. Akad. Nauk SSSR, 173, No. 4 (1967).
2. A. M. Degtyarev, L. A. Zaklyaz'minskii, et al., "Growth of finite local perturbations in electrical conductivity in a flow of weakly conducting gas in the presence of a magnetic field," Teplofiz. Vys. Temp., 7, No. 3 (1969).
3. David A. Oliver, "Time dependent phenomena in magnetohydrodynamic generators," in: Papers from the First Soviet-American Colloquium on MHD Energy Conversion, Izd. IVT Akad. Nauk SSSR, Moscow (1974).
4. V. S. Slavin, V. S. Sokolov, and V. A. Derevyanko, "A highly efficient MHD generator using gas-plasma flows with large-scale inhomogeneities," in: Thermophysical Aspects of Direct Energy Conversion [in Russian], Naukova Dumka, Kiev (1979).
5. V. A. Derevyanko, V. S. Slavin, and V. S. Sokolov, "A magnetohydrodynamic generator of electrical energy based on gasification products from brown coals," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1980).
6. N. V. Sosnin and A. P. Favorskii, Steady-State Magnetohydrodynamic Structures in a T Layer [in Russian], No. 64, Preprint IPM Akad. Nauk SSSR (1976).
7. V. A. Kamenshchikov, Yu. A. Plastinin, et al., Radiation Parameters of Gases at High Temperatures [in Russian], Mashinostroenie, Moscow (1971).
8. G. W. Sutton and A. Sherman, Engineering Magnetohydrodynamics, McGraw-Hill (1965).